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The importance of contact ratios

How to calculate the contact ratios for various styles of gearing.

It is expected that when we flip the switch on a device that it will power up and some motion will occur. In the example of the desktop printer that we all have in our home office, we expect it to go through a warmup cycle that involves the device moving all of its internal parts in preparation of printing your document or image file. This moving occurs because the motor drives the gears, which, in turn, manipulate the working of the printer. If the motor is energized but the gears do not mesh, the printer cannot perform to your expectations. One of the driving factors in establishing the amount of load that a gear pair can transmit is the contact ratio.

The contact ratio is the numerical determination of the number of teeth on each gear when any gear set is in mesh. It accounts for the teeth that are sliding into the mesh, the teeth that are sliding out of the mesh, and those that are in full contact when the gears are engaged.

The transverse contact ratio (ε_α) for spur gears is calculated using the following formula:

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_b}{\pi m \cos \alpha_0}$$

where: r_k = tip diameter (mm)
r_g = reference radius (mm)
a = center distance (mm)
α_b = working pressure angle (degrees)
α_0 = reference pressure angle (degrees)
m = module

When the working pressure angle is set to 20 degrees and module is set to 1, the values are calculated in Table 1.

The transverse contact ratio (ε_α) for helical gears is calculated using the following formula:

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_{bs}}{\pi m_s \cos \alpha_s}$$

where: r_k = tip diameter (mm)
r_g = reference radius (mm)
a = center distance (mm)
α_{bs} = transverse working pressure angle (degrees)
α_s = reference transverse pressure angle (degrees)
m_s = transverse module

The transverse contact ratio (ε_α) for straight tooth bevel gears is calculated using the following formula:

$$\varepsilon_\alpha = \frac{\sqrt{R_{vk1}^2 - R_{vg1}^2} + \sqrt{R_{vk2}^2 - R_{vg2}^2} - (R_{v1} + R_{v2}) \sin \alpha_0}{\pi m \cos \alpha_0}$$

where: R_{vk} = tip diameter on the back cone for an equivalent spur gear (mm)

$$R_{vk} = R_v + h_k = r_0 \sec\delta_0 + h_k$$

R_{vg} = reference radius on the back cone for an equivalent spur gear (mm)

$$R_{vg} = R_v \cos\alpha_0 = r_0 \sec\delta_0 \cos\alpha_0$$

R_v = back cone distance (mm)

$$R_v = r_0 \sec\delta_0$$

r_0 = pitch radius (mm)

$$r_0 = 0.5zm$$

δ_0 = reference cone angle (degrees)
h_k = addendum at outer end (mm)
α_0 = reference pressure angle (degrees)
m = module

When the reference pressure angle is set to 20 degrees and module is set to 1, the values are calculated for straight tooth bevel gears produced in the Gleason® system in Table 2.

When the reference pressure angle is set to 20 degrees and module is set to 1, the values are calculated for straight tooth bevel gears produced in the standard system in Table 3.

The transverse contact ratio (ε_α) for spiral tooth bevel gears is calculated using the following formula:

$$\varepsilon_\alpha = \frac{\sqrt{R_{vk1}^2 - R_{vg1}^2} + \sqrt{R_{vk2}^2 - R_{vg2}^2} - (R_{v1} + R_{v2}) \sin \alpha_s}{\pi m \cos \alpha_s}$$

where: R_{vk} = tip diameter on the back cone for an equivalent spur gear (mm)

$$R_{vk} = R_v + h_k = r_0 \sec\delta_0 + h_k$$

R_{vg} = reference radius on the back cone for an equivalent spur gear (mm)

$$R_{vg} = R_v \cos\alpha_0 = r_0 \sec\delta_0 \cos\alpha_0$$

R_v = back cone distance (mm)

$$R_v = r_0 \sec\delta_0$$

r_0 = pitch radius (mm)

$$r_0 = 0.5zm$$

δ_0 = reference cone angle (degrees)
h_k = addendum at outer end (mm)
α_s = mean transverse pressure angle (degrees)

$$\alpha_s = \tan^{-1} (\tan\alpha_n / \cos\beta_m)$$

m = module
α_n = reference normal pressure angle (degrees)
β_m = mean spiral angle (degrees)

When the reference pressure angle is set to 20 degrees, module is set to 1 and the spiral angle is set to 35 degrees, the values are calculated for spiral tooth bevel gears produced in the Gleason® system in Table 4.

Using the tables and formulas, you will be able to determine the proper contact ratio for most styles of gearing.

Transverse contact ratio of standard spur gears, ε_α

($\alpha_0 = 20°$)

Table 1

No. of teeth	17	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120
17	1.514																			
20	1.535	1.557																		
25	1.563	1.584	1.612																	
30	1.584	1.605	1.633	1.654																
35	1.603	1.622	1.649	1.670	1.687															
40	1.614	1.635	1.663	1.684	1.700	1.714														
45	1.625	1.646	1.674	1.695	1.711	1.725	1.736													
50	1.634	1.656	1.683	1.704	1.721	1.734	1.745	1.755												
55	1.642	1.664	1.691	1.712	1.729	1.742	1.753	1.763	1.771											
60	1.649	1.671	1.698	1.719	1.736	1.749	1.760	1.770	1.778	1.785										
65	1.655	1.677	1.704	1.725	1.742	1.755	1.766	1.776	1.784	1.791	1.797									
70	1.661	1.682	1.710	1.731	1.747	1.761	1.772	1.781	1.789	1.796	1.802	1.808								
75	1.666	1.687	1.714	1.735	1.752	1.765	1.777	1.786	1.794	1.801	1.807	1.812	1.817							
80	1.670	1.691	1.719	1.740	1.756	1.770	1.781	1.790	1.798	1.805	1.811	1.817	1.821	1.826						
85	1.674	1.695	1.723	1.743	1.760	1.773	1.785	1.794	1.802	1.809	1.815	1.821	1.825	1.830	1.833					
90	1.677	1.699	1.726	1.747	1.764	1.777	1.788	1.798	1.806	1.813	1.819	1.824	1.829	1.833	1.837	1.840				
95	1.681	1.702	1.729	1.750	1.767	1.780	1.791	1.801	1.809	1.816	1.822	1.827	1.832	1.836	1.840	1.844	1.847			
100	1.683	1.705	1.732	1.753	1.770	1.783	1.794	1.804	1.812	1.819	1.825	1.830	1.835	1.839	1.843	1.846	1.850	1.853		
110	1.688	1.710	1.737	1.758	1.775	1.788	1.799	1.809	1.817	1.824	1.830	1.835	1.840	1.844	1.848	1.852	1.855	1.858	1.863	
120	1.693	1.714	1.742	1.762	1.779	1.792	1.804	1.813	1.821	1.828	1.834	1.840	1.844	1.849	1.852	1.856	1.859	1.862	1.867	1.871
RACK	1.748	1.769	1.797	1.817	1.834	1.847	1.859	1.868	1.876	1.883	1.889	1.894	1.899	1.903	1.907	1.911	1.914	1.917	1.922	1.926

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_b}{\pi m \cos \alpha_o}$$

The transverse contact ratio for Gleason's straight bevel gear, ε_α ($\Sigma = 90°$、$\alpha_0 = 20°$)

Table 2

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.514										
15	1.529	1.572									
16	1.529	1.578	1.588								
18	1.528	1.584	1.597	1.616							
20	1.525	1.584	1.599	1.624	1.640						
25	1.518	1.577	1.595	1.625	1.650	1.689					
30	1.512	1.570	1.587	1.618	1.645	1.697	1.725				
36	1.508	1.563	1.579	1.609	1.637	1.692	1.732	1.758			
40	1.506	1.559	1.575	1.605	1.632	1.688	1.730	1.763	1.775		
45	1.503	1.556	1.571	1.600	1.626	1.681	1.725	1.763	1.781	1.794	
60	1.500	1.549	1.564	1.591	1.615	1.668	1.710	1.751	1.773	1.796	1.833

The transverse contact ratio for standard straight bevel gear, ε_α ($\Sigma = 90°$、$\alpha_0 = 20°$)

Table 3

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.514										
15	1.545	1.572									
16	1.554	1.580	1.588								
18	1.571	1.595	1.602	1.616							
20	1.585	1.608	1.615	1.628	1.640						
25	1.614	1.636	1.643	1.655	1.666	1.689					
30	1.634	1.656	1.663	1.675	1.685	1.707	1.725				
36	1.651	1.674	1.681	1.692	1.703	1.725	1.742	1.758			
40	1.659	1.683	1.689	1.702	1.712	1.734	1.751	1.767	1.775		
45	1.666	1.691	1.698	1.711	1.721	1.743	1.760	1.776	1.785	1.794	
60	1.680	1.707	1.714	1.728	1.739	1.762	1.780	1.796	1.804	1.813	1.833

The transverse contact ratio for Gleason's spiral bevel gear, ε_α ($\Sigma = 90°$、$\alpha_0 = 20°$、$\beta_m = 35°$)

Table 4

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.221										
15	1.228	1.254									
16	1.227	1.258	1.264								
18	1.225	1.260	1.269	1.280							
20	1.221	1.259	1.269	1.284	1.293						
25	1.214	1.253	1.263	1.282	1.297	1.319					
30	1.209	1.246	1.257	1.276	1.293	1.323	1.338				
36	1.204	1.240	1.251	1.270	1.286	1.319	1.341	1.355			
40	1.202	1.238	1.248	1.266	1.283	1.316	1.340	1.358	1.364		
45	1.201	1.235	1.245	1.263	1.279	1.312	1.336	1.357	1.366	1.373	
60	1.197	1.230	1.239	1.256	1.271	1.303	1.327	1.349	1.361	1.373	1.392

ABOUT THE AUTHOR

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